Random and Mixed-effects Modeling
Overview

• Effect-size estimates
• Random-effects model
• Mixed model
Effect sizes

Suppose we have computed effect-size estimates from $k$ studies, we will call them

$T_1, T_2, \ldots, T_k$

Call their variances (squares of their SE’s)

$v_1, v_2, \ldots, v_k$

Call the population effect sizes

$\theta_1, \theta_2, \ldots, \theta_k$
When is the random-effects model appropriate?

If all population parameters are equal \((\theta_i = \theta)\), we have the fixed-effects model

\[ T_i = \theta + e_i \quad \text{for } i = 1 \text{ to } k. \]

All studies are modeled as having the same effect \(\theta\).
When is the random-effects model appropriate?

Suppose that a test of homogeneity has indicated more between-studies variation than would be expected due simply to sampling error. Then we have the random-effects model

\[ T_i = \theta_i + e_i \quad \text{for } i = 1 \text{ to } k. \]

Each study has its own population effect, \( \theta_i \).
When is the random-effects model appropriate?

This model can be written in one more form:

\[ T_i = \theta_i + e_i \quad \text{for } i = 1 \text{ to } k. \]

\[ T_i = \mu_\theta + u_i + e_i \quad \text{for } i = 1 \text{ to } k, \]

where \( \theta_i = \mu_\theta + u_i \)

We have replaced the unique \( \theta_i \) with an average effect \( \mu_\theta \) plus a component \( u_i \) representing between-studies variation.
When is the random-effects model appropriate?

Suppose that we have a predictor that explains some between-studies variation. Then we have the mixed-effects model

$$T_i = \beta X_i + u_i + e_i \quad \text{for } i = 1 \text{ to } k.$$  

Observed study outcome | Population parameter predicted by $X$ | Residual deviation due to sampling and other error
---|---|---

Each study has its own population effect, $\theta_i$. 
Random-effects Model
Random-effects analyses

The bars represent four true $\theta_i$ values, and the histogram shows the observed effect sizes. $\mu_\theta$ would be the mean of the $\theta$s.

$$T_i = \theta_i + e_i$$
In random-effects analyses, the goal is to estimate:

- the mean population effect size $\mu_\theta$ of the populations from which the observed studies are a sample (central tendency of the I bars)

and

- between-study variation in effect sizes (in the $\theta_i$ or $u_i$), which has an impact on weights and the uncertainty of this mean. This is often called $\tau^2$ or $\sigma^2_\theta$. It is the variation in the I bars.
The random-effects model

We will add the between-studies variance $\hat{\sigma}_\theta^2$ to each study’s $\nu_i$ and use weighted least squares (WLS) estimation with new random-effects weights.

The new variances for each study will be larger than the fixed-effects variances.

Because of this, sometimes means that were significant under the fixed-effects model may no longer be significant under the random model.
Let us consider an example based on the teacher expectancy data from Raudenbush (1984).

The 19 effects compare IQ test scores for students randomly labeled as “bloomers” to other unlabeled students.
Random-effects analyses: Example

The histogram shows that effects vary from about -0.5 to above 1 standard deviation.
Random-effects analyses: Example

The 95% CI plot shows a fair amount of variation. The “quick and dirty” test of drawing a line across the plot shows no line can cross all the CIs.
Random-effects analyses: Example

The overall test of homogeneity, obtained from the SPSS GLM menu, shows significant between studies variation. This is a chi-square test with 18 df, and \( p = .007 \).

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>.000(^a)</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>2.735</td>
<td>1</td>
<td>2.735</td>
<td>1.374</td>
<td>.256</td>
</tr>
<tr>
<td>Error</td>
<td>35.825</td>
<td>18</td>
<td>1.990</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>38.561</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>35.825</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. R Squared = .000 (Adjusted R Squared = .000)
b. Weighted Least Squares Regression - Weighted by w
Estimating the variance of $\theta_1, \theta_2, \ldots, \theta_k$

One estimator (SVAR on slide 18) is

$$\hat{\tau}^2 = \hat{\sigma}_\theta^2 = S^2_T - \bar{v} = .129 - .0484 = .0806$$

where $S^2_T$ is the simple variance of the observed effects and $\bar{v}$ is the mean of the “known” fixed-effects variances $v_i$, $\bar{v} = \sum v_i / k$

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>19</td>
<td>-0.32</td>
<td>1.18</td>
<td>0.1637</td>
<td>0.35887</td>
<td>0.129</td>
</tr>
<tr>
<td>V</td>
<td>19</td>
<td>0.01</td>
<td>0.14</td>
<td>0.0484</td>
<td>0.04063</td>
<td>0.002</td>
</tr>
<tr>
<td>Valid N (listwise)</td>
<td>19</td>
<td>0.01</td>
<td>0.14</td>
<td>0.0484</td>
<td>0.04063</td>
<td>0.002</td>
</tr>
</tbody>
</table>
Another estimator of $\tau^2$ or $\sigma^2_\theta$ is

$$\hat{\tau}^2 = \frac{[Q - (k - 1)]}{c}$$

where

$$c = \left( \begin{array}{c} \frac{k}{\sum_{i=1}^{k} w_i^2} \\ \frac{\sum_{i=1}^{k} w_i}{\sum_{i=1}^{k} w_i} \\
\end{array} \right)$$

whenever the estimate is greater than 0, and 0 otherwise. This is called QVVAR on slide 18.
Random-effects analyses: Example

We can also get the Q test and variances from SAS. Here we see results of estimation of the simple random-effects variance for the data – the more conservative is .08:

### Random-effects analyses: Example

<table>
<thead>
<tr>
<th>OBS</th>
<th>K</th>
<th>Q</th>
<th>P</th>
<th>LL_T_DOT</th>
<th>T_DOT</th>
<th>UL_T_DOT</th>
<th>V_T_DOT</th>
<th>SE_T_DOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19</td>
<td>85.8254</td>
<td>.0074284</td>
<td>-0.011168</td>
<td>0.060343</td>
<td>0.13185</td>
<td>.0013312</td>
<td>0.036485</td>
</tr>
</tbody>
</table>

Fixed-Effects Effect Size Analysis (Exercise 2), tchrexp.dat

<table>
<thead>
<tr>
<th>OBS</th>
<th>MODVAR</th>
<th>MODSD</th>
<th>QVAR</th>
<th>QSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.022398</td>
<td>0.14966</td>
<td>0.025920</td>
<td>0.16100</td>
</tr>
</tbody>
</table>

\[
\hat{\sigma}^2 \theta
\]

<table>
<thead>
<tr>
<th>OBS</th>
<th><em>TYPE</em></th>
<th><em>FREQ</em></th>
<th>K</th>
<th>SUMV</th>
<th>SUMT</th>
<th>SUMT2</th>
<th>SVAR</th>
<th>SSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>19</td>
<td>19</td>
<td>0.92009</td>
<td>3.11</td>
<td>2.8273</td>
<td>0.080366</td>
<td>0.28349</td>
</tr>
</tbody>
</table>
Random-effects analyses: Example

For the random-effects model, we need to compute an average of the differing effects.

We use a weighted mean – BUT we weight each data point by the inverse of its random-effects variance (i.e., \( w^*_i = 1/[V(T_i) + \hat{\sigma}_\theta^2] \)):

\[
T^* = \sum_{i=1}^{k} \frac{w^*_i T_i}{w^*_i} = \sum \frac{T_i}{[V(T_i) + \hat{\sigma}_\theta^2]} \cdot \frac{1/[V(T_i) + \hat{\sigma}_\theta^2]}{1}
\]
The fixed effects variance is $v$ and the random effects variance is $v_{\text{star}} = v_i + .08$.

Not only are the random effects variances larger, but they are also more equal in size.
Random-effects analyses: Example

We use the random-effects variance to compute a new mean. It is somewhat larger than the fixed effects mean. The printed SE needs to be corrected, via $SE = SE_{\text{printed}} \sqrt{\text{MSE}}$.

Here $SE = .071 \times .89 = .064$ (vs. $SE_{\text{fixed}} = .036$)

<table>
<thead>
<tr>
<th>Mean</th>
<th>Std. Error</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>.114</td>
<td>.071</td>
<td>Lower Bound</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-.035</td>
</tr>
</tbody>
</table>

a. Weighted Least Squares Regression - Weighted by wstar
If we compute the mean $\pm 1.96 \, \hat{\sigma}_\theta$ we can get a range of possible population $\theta_i$ values. Computing $0.114 \pm 1.96 \times 0.283$, 95% of the $\theta_i$ values are estimated to be between -.44 and .67, assuming a normal distribution of $\theta_i$s.
Mixed-effects Model
When is the mixed-effects model appropriate?

Suppose we have attempted to fit a model (either a regression model or the ANOVA-like categorical model) and the model is significant but does not explain all variation. That is, we find that both $Q_{\text{Model}}$ and $Q_{\text{Error}}$ are significant.

We want to keep the predictors that are useful, but also to account for the remaining uncertainty. In this case, the mixed-effects model can be adopted.
The components in the mixed-effects model

This terminology is different from the use of the label “mixed model” in ANOVA where sometimes this term can refer to a model with both between-subjects and within-subjects terms.

Mixed-effects models in meta-analysis contain both *fixed* and *random* components.
The components in the mixed-effects model

For the ANOVA-like model, with study $i$ from the $j$th group or category of studies, we may have

$$T_{ji} = \theta_{j.} + u_{ji} + e_{ji}$$

for study $i$ in group $j$

and for the regression model

$$T_i = \beta_0 + \beta_1 X_{1i} + ... + \beta_p X_{pi} + u_i + e_i$$

Those familiar with hierarchical modeling will recognize these as hierarchical linear models.
The components in the mixed-effect model

In the ANOVA-like mixed model, we have

\[ T_{ji} = \theta_j + u_{ji} + e_{ji} \]

for study \( i \) in group \( j \)

For the regression model with \( p \) predictors

\[ T_i = \beta_0 + \beta_1 X_{1i} + \ldots + \beta_p X_{pi} + u_i + e_i \]
We will need to estimate the variation of the $u_i$ error terms in the mixed-effects model. We will denote this variance as $\hat{\sigma}^2_{\theta | X}$.

We don’t want to use simple random-effects estimators because they ignore the fact we have useful predictors in our model. They often will be too large.

So we need similar variances, but which estimate only what is left unexplained by the model we choose.
Estimating mixed models

• It is possible to use standard statistical packages (such as SAS or SPSS) to estimate mixed models using a multi-step process, but it can also be done in one pass with SPSS or SAS macros or by using more specialized software such as HLM or Stata.

• The metareg routine for Stata routinely estimates a mixed model, as does the V-known option of HLM.
Estimating mixed models

• In these slides we will walk through the step-by-step process using output from SAS and SPSS for the teacher expectancy data.

• An example of Stata metareg output for levels of political interest (Block & Becker, 2008) will also be shown, as will analyses for a dataset on the relationship of science teacher knowledge to student science achievement (Becker & Aloe, 2008).
The mixed-effects model: Regression model

We will begin with an ordinary least squares (OLS) regression. This means we will not use the weights typically used for meta-analysis.

We use whatever predictors we’ve found significant in a fixed-effects weighted analysis, and run an unweighted regression with them.

Then we will use the $MSE$ from the OLS regression model to compute a variance that is similar to the simple methods-of-moments between-studies variance.
The mixed-effects model: Regression model and $\hat{\sigma}^2_{\theta|X}$

The method-of-moments random-effects variance is

$$\hat{\sigma}^2_{\theta} = S_T^2 - \bar{v}$$

where $\bar{v} = \sum \nu_i / k$ is the mean fixed-effects variance. For the mixed model we compute the mixed-effects model variance

$$\hat{\sigma}^2_{\theta|X} = MSE_{OLS} - \bar{v}$$

where $MSE_{OLS}$ is the mean squared error from the OLS (unweighted) regression.
If the regression predictor is useful, $MSE_{OLS}$ will be smaller than $S_T^2$ (this is true even though we are not using the proper weighting – also note that we do not use the slopes from the OLS regression for anything!).

Therefore, since $S_T^2 > MSE_{OLS}$

then $\hat{\sigma}_\theta^2 > \hat{\sigma}_{\theta|X}^2$

So we expect the mixed-model variance to be lower than the simple random-effects variance.
The mixed-effects model: Regression model

We add the new variance $\hat{\sigma}^2_{\theta|X}$ to each study’s $\nu_i$ and use weighted least squares (WLS) regression with new mixed-model weights.

The mixed model variances for each study should also be larger than the fixed-effects variances but smaller than the random-effects values.

Because of this, predictors that were significant under the fixed-effects model may no longer be significant under the mixed model.
Thus, we have three possible weights:

**Fixed**  \( w_i = 1/v_i \)

**Random**  \( w_i^* = 1/[v_i + \hat{\sigma}_\theta^2] \)

**Mixed**  \( w_i^M = 1/[v_i + \hat{\sigma}_{\theta|X}^2] \)

The mixed-model weights \( w_i^M = 1/[v_i + \hat{\sigma}_{\theta|X}^2] \) are used instead of \( w_i \) or \( w_i^* \) to estimate the mixed-effects model.

Also there is not one unique set of mixed model weights – since the value of \( \hat{\sigma}_{\theta|X}^2 \) depends on exactly what \( X \)s are included in your model.
Example: Teacher expectancy data

We have two values of the simple random-effects variance for the data – the more conservative is .08:

<table>
<thead>
<tr>
<th>OBS</th>
<th>K</th>
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Fixed-Effects Effect Size Analysis (Exercise 2), tchrexp.dat

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<td>0.025920</td>
<td>0.16100</td>
</tr>
</tbody>
</table>

We will run the WLS and OLS regressions on the teacher expectancy data. The WLS regression model on $X=$ weeks shows a slight amount of misfit.
The relationship of “weeks” to the effects is somewhat nonlinear. The OLS “best fit” line that SPSS plots will not be correct unless all studies are equal in size.

This occurs because we need to use a weighted regression and the plotted line is unweighted.
The fixed-effects model: Estimating the line

The fixed-effects equation is obtained by using the SPSS regression menu with “w” =1/v_i clicked into the box for the “WLS weight”.

The model is

\[ d_i = 0.158 - 0.013 \text{weeks}_i + e_i \]
Here’s the regression line from the weighted analysis:

\[ d_i = 0.158 - 0.013 \text{ weeks}_i + e_i \]

Clearly the points with the large effects at weeks = 0 are not well explained by this line.
The fixed-effects model: Testing significance of \( x \)

Is the predictor “weeks” significant under the fixed effects model? If not, we do not need to proceed to a mixed model for this predictor. Here, it is significant:

\[
SS_{\text{Regression}} = Q_{\text{Model}} = 8.61 \text{ is significant with } p = 1 \text{ df.}
\]

<table>
<thead>
<tr>
<th>Model</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>8.161</td>
<td>1</td>
<td>8.161</td>
<td>5.015</td>
<td>.039a</td>
</tr>
<tr>
<td>Residual</td>
<td>27.664</td>
<td>17</td>
<td>1.627</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>35.825</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- a. Predictors: (Constant), weeks
- b. Dependent Variable: \( T \)
- c. Weighted Least Squares Regression - Weighted by \( w \)
The fixed-effects model: Testing specification

We’d like the $Q_{\text{Error}}$ to be small and not significant.

$SS_{\text{Residual}} = Q_{\text{Error}} = 27.66$ is just barely significant with $p = 17$ df. We may want to fit the mixed model because we know some studies with weeks = 0 do not fit well.

<table>
<thead>
<tr>
<th>Model</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Regression</td>
<td>8.161</td>
<td>1</td>
<td>8.161</td>
<td>5.015</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>27.664</td>
<td>17</td>
<td>1.627</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>35.825</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- a. Predictors: (Constant), weeks
- b. Dependent Variable: $T$
- c. Weighted Least Squares Regression - Weighted by $w$
The mixed-effects model: Variance estimation

We get the mixed-model variance. $MSE_{OLS}$ is .094. Also $\bar{\nu}$ is the same at .0484.

<table>
<thead>
<tr>
<th>Model</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Regression</td>
<td>.728</td>
<td>1</td>
<td>.728</td>
<td>7.786</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>1.590</td>
<td>17</td>
<td>.094</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>2.318</td>
<td>18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Predictors: (Constant), weeks
b. Dependent Variable: T

$MSE_{OLS}$

$$\hat{\sigma}^2_{\theta|X} = MSE_{OLS} - \bar{\nu} = .094 - .0484 = .0456$$

Recall the random-effects variance was larger, at .08
We add $\hat{\sigma}^2_{\theta|X} = .0456$ to each study’s variance and run a weighted mixed-model regression. The model is still significant but $Q_{\text{Error}}$ has dropped since we are accounting for $\hat{\sigma}^2_{\theta|X}$.

$$Q_{\text{Model}} = 5.56 \text{ with } df = 1, \quad Q_{\text{Error}} \text{ is NS.}$$

<table>
<thead>
<tr>
<th>Model</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Regression</td>
<td>5.564</td>
<td>1</td>
<td>5.564</td>
<td>7.031</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>13.452</td>
<td>17</td>
<td>.791</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>19.015</td>
<td>18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Predictors: (Constant), weeks
b. Dependent Variable: Tb.
c. Weighted Least Squares Regression - Weighted by wmc
The mixed-effects model: Estimating the line

Recall that the FE model was

\[ T_i = 0.158 - 0.013 \text{weeks}_i + e_i \]

The mixed model is quite similar

\[ T_i = 0.238 - 0.019 \text{weeks}_i + u_i + e_i \]

<table>
<thead>
<tr>
<th>Model</th>
<th>Unstandardized Coefficients</th>
<th>Standardized Coefficients</th>
<th>t</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(Constant)</td>
<td>.238</td>
<td>.078</td>
<td>3.057</td>
</tr>
<tr>
<td></td>
<td>weeks</td>
<td>-.019</td>
<td>.007</td>
<td>-.541</td>
</tr>
</tbody>
</table>

- \( a \). Dependent Variable: \( T \)
- \( b \). Weighted Least Squares Regression - Weighted by \( \text{wm} \)
Output from a mixed model in Stata

This model from Block and Becker (2008) predicts the proportion of people expressing any level of political interest (called any_x) from the year a survey was conducted.

There appear to be increasing levels of interest.
Stata produces this output from its routine called meta-reg

```
.metareg any_4 yr40, wsse(seany4) bsest(mm)
```

**Output from a mixed model in Stata**

- Number of studies = 25
- Q (23 df) = 301.155
- Prob > Q = 0.000
- I-squared = 0.924
- tau2 = 0.0010

| any_4 | Coef.  | Std. Err. | t     | P>|t| | [95% Conf. Interval] |
|-------|--------|-----------|-------|-----|----------------------|
| yr40  | 0.0016399 | 0.0006019 | 2.72  | 0.012 | 0.0003948 - 0.002885 |
| _cons | 0.7976401 | 0.0296878 | 26.87 | 0.000 | 0.7362262 - 0.859054 |

The mixed model is

\[ p_i = 0.798 + 0.0016 \text{yr40}_i + u_i + e_i \]

with yr40 = year - 1940
The mixed-effects model: ANOVA-like categorical model

The ANOVA model is more complicated than the regression model because there are several ways we can have a mixed model.

For instance, it may be that only one category or group has studies that are heterogeneous.

Another possibility is that all groups are heterogeneous, but to different degrees. A final case is that all are heterogeneous to the same degree.
The mixed-effects model: \( RSSVAR \) in the ANOVA-like model

The simplest analysis may be to compute the RE variance within each category or group (e.g., using the \( \hat{\sigma}_\theta^2 \) formulas—let’s say we get \( \hat{\sigma}_\theta^2 \) for group \( j \)) and add a different value to each study’s \( \nu_{ij} \) depending on which group the study is in.

This approach allows all groups to vary (or not) and assigns different variances for the \( u_i \) values in each subset.
A weakness with estimating the within group variance for each separate group of studies is that some groups may be very small and $\hat{\sigma}^2_{\theta}$ and $\hat{\sigma}^2_{\theta|X}$ are poorly estimated with few studies.

If we are willing to assume that all groups are heterogeneous to the same degree, we can use a parallel approach to the regression mixed model.
The mixed-effects model: \( \hat{\sigma}^2_{\theta|X} \) in the ANOVA-like model

We run an unweighted ANOVA to get a \( MSW \) (analogous to the \( MSE_{OLS} \)) and compute the \( \hat{\sigma}^2_{\theta|X} \) in the way described above.

This approach has the weakness that it may make some studies have larger (or smaller) variances than are really appropriate, if the groups are not equally heterogeneous.
Example: Teacher expectancy data

We can use “weeks” as a categorical variable if we categorize studies as having 0, 1, 2 or 3+ weeks of exposure. We call it “weekcat”.

Although the ANOVA model does not misfit for the TE data, let’s consider how the analyses would work. If we choose to assume equal heterogeneity, we’d run an unweighted ANOVA and get the $MS_W$, which is .077.

<table>
<thead>
<tr>
<th></th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>1.164</td>
<td>3</td>
<td>.388</td>
<td>5.039</td>
<td>.013</td>
</tr>
<tr>
<td>Within Groups</td>
<td>1.155</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2.318</td>
<td>18</td>
<td>.077</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example: Teacher expectancy data

We compute the common within groups variance as we did above:

\[
\hat{\sigma}^2_{\theta|X} = 0.077 - 0.0484 = 0.0286
\]

The value 0.0286 would be added to every study’s variance and we would run a weighted ANOVA using the new weights = 1/[\nu_i + 0.0286].
Example: Teacher expectancy data

Alternately we may wish to examine each group of studies, to see if some groups are more heterogeneous than others.

From our initial plot of the weeks – effect size relationship we may recall that effects with no weeks of exposure (weeks = 0) seemed more variable.

If we find different heterogeneity in each group, we’d add different variances $\hat{\sigma}^2_{\theta|Xj}$ to each study’s $\nu_i$ in group $j$. 
### Example: Teacher expectancy data

Here are values of $S_T^2$ and $\hat{\Sigma}_{\theta j}$ for each `weekcat` group. $\hat{\sigma}_{\theta j}^2$ values are shown at the right – two values are negative and would be set to 0. For studies with `weekcat` = 2 or 3 we can use $w_i = 1/v_i$, but we’d use $w_i = 1/[v_i + .116]$ for `weekcat` = 1, etc. If some groups are more variable this approach may be best.

#### Descriptive Statistics

<table>
<thead>
<tr>
<th><code>weekcat</code></th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>T</td>
<td>5</td>
<td>.4860</td>
<td>.41107</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>5</td>
<td>.0886</td>
<td>.05035</td>
</tr>
<tr>
<td></td>
<td>Valid N (listwise)</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>T</td>
<td>3</td>
<td>.3600</td>
<td>.41328</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>3</td>
<td>.0553</td>
<td>.03279</td>
</tr>
<tr>
<td></td>
<td>Valid N (listwise)</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>3</td>
<td>.1100</td>
<td>.10583</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>3</td>
<td>.0362</td>
<td>.04163</td>
</tr>
<tr>
<td></td>
<td>Valid N (listwise)</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>T</td>
<td>8</td>
<td>-.0913</td>
<td>.12800</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>8</td>
<td>.0253</td>
<td>.01205</td>
</tr>
<tr>
<td></td>
<td>Valid N (listwise)</td>
<td>8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\hat{\Sigma}_{\theta j}$

<table>
<thead>
<tr>
<th>Weekcat</th>
<th>$S_T^2$</th>
<th>$\hat{\sigma}_{\theta j}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.169</td>
<td>.089 = .080</td>
</tr>
<tr>
<td>1</td>
<td>.171</td>
<td>.055 = .116</td>
</tr>
<tr>
<td>2</td>
<td>.011</td>
<td>.036 = -.025 = 0</td>
</tr>
<tr>
<td>3</td>
<td>.016</td>
<td>.025 = -.009 = 0</td>
</tr>
</tbody>
</table>
Thanks are due to the following institutions and individuals

- Funder: Norwegian Knowledge Centre for the Health Sciences
- Materials contributors: Betsy Becker, Harris Cooper, Larry Hedges, Mark Lipsey, Therese Pigott, Hannah Rothstein, Will Shadish, Jeff Valentine, David Wilson
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